Impact of fund size on hedge fund performance

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Abstract This paper investigates whether the increase in assets flowing into the hedge fund industry diminishes returns and, in particular, whether larger hedge funds underperform smaller hedge funds, as is often conjectured, owing to limited capacity in certain hedge fund strategies. The impact of fund sizes is analysed with respect to fund returns, standard deviations, Sharpe ratios and alphas derived from a multi-asset class factor model.

Keywords: *hedge funds, performance measurement, size*

Introduction

In recent years, hedge funds have gained widespread acceptance owing to their interesting risk–return characteristics and their low correlations to traditional asset classes. Many studies have investigated the factors affecting hedge fund returns. The factors vary between the different hedge fund strategies and, additionally, the variety of strategies is increasing in the hedge fund industry. At the same time, the hedge fund industry has experienced impressive growth such that capacity is becoming a serious issue not only for large hedge fund investors looking for investment opportunities to employ the capital, but also for hedge fund investors looking for good hedge funds that are open for investment. Increasing efficiency of financial markets results in decreasing arbitrage opportunities, which are the primary source for returns of some hedge fund strategies. This paper investigates whether an increasing asset base of hedge funds is diluting performance.

On the one hand, it can be conjectured that small hedge funds are underperforming larger hedge funds owing to a higher expense ratio. On the other hand, many investment professionals argue that smaller funds are outperforming larger funds owing to a higher risk appetite or their enhanced flexibility to concentrate the capital under management on their best investment ideas. The best ten investment ideas of a hedge fund manager are generally better than the best 100 ideas. Smaller hedge funds are also more nimble and therefore more liquid, owing to smaller position sizes.

Large funds may face difficulties in liquidating their positions in difficult market environments.

Several studies touch on the subject with varying results. While the effect of fund size on performance is one of the largest concerns in the hedge fund industry, it has received little research attention in studies exclusively focusing on this subject. This paper attempts to fill this gap by evaluating the relationship between fund size and performance from different angles. The study is supported by empirical evidence based on a large data sample of hedge fund returns and fund sizes.

A number of different multi-asset class factor models have been used to derive alphas of hedge fund returns. In contrast to the standard asset class factor model of Sharpe (1992), this study uses excess returns to derive alphas from the factor models in order to investigate the impact of fund sizes. The empirical study provides some evidence for a positive relationship between fund sizes and hedge fund performance.

The structure of the paper is the following: it starts with a literature overview, followed by a description of the dataset used for the analysis. Next, the methodology is introduced and an empirical analysis concerning the impact of fund sizes on hedge fund returns, standard deviations, Sharpe ratios and alphas is conducted.

Literature overview

Several authors propose factor models to explain hedge fund returns. Fung and Hsieh (1997) employ Sharpe's (1992) model and use a principal component analysis to explain hedge fund returns. Agarwal and Naik (2000) develop a factor model and point out the non-linear option-like exposures of various hedge fund strategies. An interesting approach to overcoming the short history of hedge fund returns has been proposed by Agarwal and Naik (2002). The authors use the underlying risk factors estimated with a multi-factor model to simulate the effects of the major stock market crises of 1929 and 1987 on hedge fund returns. Schneeweis *et al.* (2003) investigate the differences between a single-factor and a multi-factor model to explain hedge fund strategy returns. Brealey and Kaplanis (2001) investigate changes in factor exposures to explain hedge fund returns over time.

Clark (2003) provides a comprehensive study about the relationship of fund assets to performance for mutual funds and concludes that no significant return differences can be found between small and large funds on a variety of holding periods from 1991 to 2001. Herzberg and Mozes (2003) investigate the impact of several factors on hedge fund performance and find that smaller hedge funds display a performance that is better but barely significantly so compared with larger funds, while the difference is significantly positive regarding Sharpe ratios.

Hedges (2003) shows that smaller funds outperform larger funds, but finds that mid-sized funds perform the worst. This phenomenon is explained by the concept of mid-life crises for hedge fund managers.

Gregoriou and Rouah (2003) find no correlation between the size of hedge funds and their performance. The relationship is tested with Pearson's correlation coefficient and Spearman's rank correlation from January 1994 to December 1999. Using the geometric mean, the Sharpe ratio and the Treynor ratio, the correlations are not statistically significant. The sample is composed of 204 hedge funds and 72 funds of hedge funds, and is significantly smaller than in the present study and therefore not necessarily representative of the hedge fund industry.

Edwards and Caglayan (2001) argue that hedge fund performance increases at a declining rate as fund sizes increase. The authors derive six-factor alphas from a similar framework to that of Fama and French (1993, 1996). The six-factor alphas are then regressed on five variables: size, the reciprocal of size to capture non-linearity in the size–performance relationship, age and both management and incentive fees. Both size variables are statistically significant for all hedge funds and for all investment styles except 'global macro' and 'global'. A positive coefficient on the size variable together with a negative coefficient on the size reciprocal variable indicate that hedge fund performance increases at a declining rate as fund sizes increase.

Liang (1999) investigates the impact of fund characteristics with a cross-sectional regression and finds a significant positive relationship between fund assets and performance. The assets of the funds are taken only from one point in time at the end of the period. Therefore the result may simply suggest that successful funds attract more money over time and therefore have a positive correlation to past performance. The study therefore does not necessarily measure the impact of fund assets on performance, but the impact of performance on fund assets. The dataset used contains only 385 funds investigated over a three-year time horizon from January 1994 to December 1996.

Amenc and Martellini (2003) support the view by investigating two equally sized groups with large and small funds. For each group, the average alpha is computed based on a number of different models, such as the standard capital asset pricing model (CAPM), an adjusted CAPM for the presence of stale prices and an implicit factor model extracted from a principal component analysis. For all models, the mean alpha for large funds exceeds the mean alpha for small funds. The separation of the data into small and large funds is simplistic and not sufficient to measure the relationship between fund sizes and performance.

A similar approach is chosen by Kazemi and Schneeweis (2003). At the beginning of each year, funds within each style are assigned either a large or a small subgroup, depending on the size of assets under management. The authors find that large or small funds do not uniformly outperform the other group. The study contains only 15–30 hedge funds in each subgroup.

Getmansky (2004) analyses the effects of fund- and strategy-specific factors on the life cycles of hedge funds and confirms that better–performing funds are more likely to attract assets than poorly performing funds. The performance-asset size relationship takes on different functional forms for different strategies. For instance, for illiquid strategies such as 'emerging markets' and 'convertible arbitrage', the relationship is concave, so that the top performing funds do not grow proportionally as much as the average fund in the market. The use of quadratic regressions raises the question of data fitting. The use of the relationship between fund sizes and the performance of individual strategies is limited to the relatively low number of funds per strategy.

Goetzmann *et al.* (2003) examine the relationship between fund flows and past performance for hedge funds by regressing net fund growth on lagged return in cross section. The differential response of new money to past returns is examined via a piecewise linear regression. The authors find that new

Table 1 Data quality in the TASS database[a]

	In funds	In %
Missing returns		
Funds with at least 1 missing return	675	11.83
More than 1%	559	9.79
More than 3%	214	3.75
More than 5%	117	2.05
More than 10%	36	0.63
Missing assets		
At least 1 datapoint	3122	54.70
More than 1%	3064	53.69
More than 3%	2558	44.82
More than 5%	2228	39.04
More than 10%	1763	30.89
More than 20%	1368	23.97

[a]A large number of funds have missing data points in their time series of assets under management. The data quality is higher for return data.
Source: TASS database.

money responds by flowing out of the poorest performers.

Dataset

Three databases from TASS, one of today's largest commercial hedge fund data providers, are combined. Liang (2000, 2003) investigated the data quality of various hedge fund data providers and concluded that the TASS database has a high data quality in comparison with other commercial database providers. Many empirical studies are based on the TASS database. TASS maintains four separate databases: a hedge fund database with 'living' hedge funds; one 'graveyard' database with funds that stopped reporting; one CTA database with 'living' hedge funds; and one 'CTA graveyard'. The combination of the four databases containing 'living' and 'dead' funds allows us to derive the survivorship bias.[1] TASS started to build their databases in 1993/1994. The data prior to 1994 have been backfilled by hedge fund managers starting to report in 1994 or later. Therefore, data prior to 1994 contain a number of biases and have not been used for this analysis.

All four TASS databases combined contain a total of 7,588 funds: 3,619 funds in the hedge fund database; 2,123 funds in the hedge fund graveyard database; 493 funds in the CTA database; and 1,353 funds in the CTA graveyard database. Owing to poor data quality or double counting, 566 funds are eliminated from the data sample. Of the remaining funds, 1,315 are classified as funds of hedge funds and have been excluded from the hedge fund data sample. For the remaining 5,707 funds, the inconsistencies found are shown in Table 1. The findings confirm the concerns of other performance studies that the asset data of hedge funds are very often incomplete. The inconsistencies in the asset development can partly be explained by the fact that some hedge funds have been reporting their assets only on a quarterly, semi-annual or annual basis, particularly in their early years of reporting.

On the one hand, funds with incomplete data need to be eliminated in order to avoid any additional biases in the data sample. On the other hand, one does not want to lose too many funds for the analysis, since the sample would then be less representative for the hedge fund universe. It is therefore necessary to

Table 2 Data quality of dataset with 4,014 funds used in the empirical part of the study

	In funds	In %
Missing returns		
Funds with at least 1 missing return	481	11.98
More than 1%	391	9.74
More than 3%	130	3.24
More than 5%	59	1.47
More than 10%	0	0.00
Missing assets		
At least 1 datapoint	1739	43.32
More than 1%	1681	41.88
More than 3%	1176	29.30
More than 5%	850	21.18
More than 10%	388	9.67
More than 20%	0	0.00

Source: TASS database. 4,014 hedge funds are used for the empirical analysis.

find a compromise between data quality and data quantity. It was decided to eliminate all funds with more than 10 per cent of data points missing in return data[2] or more than 20 per cent of data points missing in asset data.[3] The initial sample is based on 5,707 hedge funds in March 2005. With the adjustment, 1,693 funds with assets of US$81bn in March 2005 are excluded. Missing data points in assets under management are the main restriction. Finally, a sample of 4,014 funds was used with assets of US$363bn in March 2005. The data quality of the sample used in the performance study is illustrated in Table 2.

Methodology

The first step was to analyse the difference between asset-weighted and equally weighted returns. The difference in the survivorship bias using equally and asset weighted returns is also investigated. Logarithmic data are used for the return analysis.

The second step uses an asset class factor model to explain excess returns.[4] Therefore 18 asset class factors are defined. The 18 asset class factors are the Dow Jones AIG Commodity Index, the Gold Index, the Goldman Sachs Commodity Index, the IPE Brent Crude Oil Index, the JP Morgan Government Bond Index, the Lehman Aggregate Bond Index, the Lehman High Yield Index, the Merrill Lynch Treasury 10+ Year Index, the MSCI Europe, the MSCI Japan, the MSCI Pacific, the MSCI World, the NASDAQ Composite Index, the Wilshire Growth Index, the Wilshire Real Estate Investment Trust Index, the Wilshire Small Cap 1750 Index, the Wilshire Value Index, and the Chicago Board Options Exchange SPX Volatility Index. The factor model is similar to the Sharpe's (1992) 'style regression', with the difference that the risk-free rate is distracted and therefore the excess return is used as the dependent variable. The equation

$$r_t - r_{ft} = Alpha + \sum_{k=1}^{n} \beta_k x_{kt} + \varepsilon_t \qquad (1)$$

with k factors and the factor loadings β_k is used.[5] The least square method is used in the regressions. A factor structure is assumed for returns according to the arbitrage pricing theory. In order to facilitate the interpretations of the results, the number of factors is reduced for further analysis.

The next step breaks the sample

according to the fund sizes into 100 percentiles. The average fund size and the average returns are calculated for each percentile *i*. Monthly data are used to conduct the analysis. The average annualised returns for the 100 percentiles are then regressed on the natural logarithms of the average asset sizes. Therefore, a linear regression is specified of the form

$$r_i = \alpha_i + \beta_1 \log(Assets_i) + \varepsilon_i \quad (2)$$

and a quadratic regression of the form

$$r_i = \alpha_i + \beta_1 \log(Assets_i) + \beta_2 \log(Assets_i)^2 + \varepsilon_i \quad (3)$$

Similarly, standard deviations and Sharpe ratios are also regressed on the logarithm of the average assets. The standard deviations and Sharpe ratios are referring to percentiles and not to individual funds. Each percentile can be considered as a portfolio of funds with similar fund sizes.

$$\sigma_i = \alpha_i + \beta_1 \log(Assets_i) + \varepsilon_i \quad (4)$$

$$\sigma_i = \alpha_i + \beta_1 \log(Assets_i) + \beta_2 \log(Assets_i)^2 + \varepsilon_i \quad (5)$$

$$SR_i = \alpha_i + \beta_1 \log(Assets_i) + \varepsilon_i \quad (6)$$

$$SR_i = \alpha_i + \beta_1 \log(Assets_i) + \beta_2 \log(Assets_i)^2 + \varepsilon_i \quad (7)$$

For the calculation of the Sharpe ratios, 90-day T-bill rates are used as the risk-free rate.

The alphas are also calculated for each individual percentile, and therefore the model described in Equation (1) is applied 100 times to derive Equation (8).

$$r_{it} - r_{ft} = Alpha_i + \sum_{k=1}^{n} \beta_{ik}(x_{ikt} - r_{ft}) + \varepsilon_{it} \quad (8)$$

The relationship is then tested between the alphas derived from the 100 factor models and the average fund sizes for the 100 percentiles with the following linear and quadratic regressions.

$$Alpha_i = \alpha_i + \beta_1 \log(Assets_i) + \varepsilon_i \quad (9)$$

$$Alpha_i = \alpha_i + \beta_1 \log(Assets_i) + \beta_2 \log(Assets_i)^2 + \varepsilon_i \quad (10)$$

Hedge fund returns

In this section, the empirical analysis is conducted, with the data sample described earlier.

Equally versus asset weighted returns

Unlike in previous studies, the focus is on the average returns achieved by hedge fund investors in contrast to the returns achieved by the average hedge fund. The difference lies in the measurement method of hedge fund returns. To the authors' knowledge, most existing studies about the performance of hedge funds use equally weighted returns to estimate returns of the unobservable hedge fund universe. One reason why most studies have focused on equally weighted hedge fund returns is the poor data quality of hedge funds' assets under management. Since the quality of available data has improved significantly in recent years, it is now feasible to calculate asset weighted returns. In this paper asset weighted returns of a large sample of hedge funds are used for performance measurement purposes. A number of index providers have developed different methodologies to benchmark hedge fund returns. Most hedge fund indices are equally weighted with the exception of the CSFB-Tremont hedge fund indices[6] and some of the MSCI hedge fund indices.

Both equally weighted and asset weighted returns for the data sample based on the TASS databases are

Table 3 Return comparison based on a sample with 4,014 hedge funds[a]

	Jan 94–Apr 05	Jan 94–Aug 99	Sep 99–Apr 05
Equally weighted returns p.a. (%)	8.42	8.98	7.86
Standard deviation p.a. (%)	5.59	6.05	5.13
Sharpe ratio	0.82	0.68	0.99
Asset weighted returns p.a. (%)	7.47	8.33	6.60
Standard deviation p.a. (%)	5.72	6.86	4.33
Sharpe ratio	0.64	0.50	0.88
Annualised differences in returns (%)	–0.96	–0.65	–1.26
Standard deviation p.a. (%)	2.97	3.85	1.72
t-statistic	–1.08	–0.40	–1.74

[a]Equally weighted and asset weighted returns are calculated for a 136-month time horizon from January 1994 to April 2005 and for two 68-month sub-periods from January 1994 to August 1999 and from September 1999 to April 2005. The significance of the return differences is tested with a *t*-statistic.

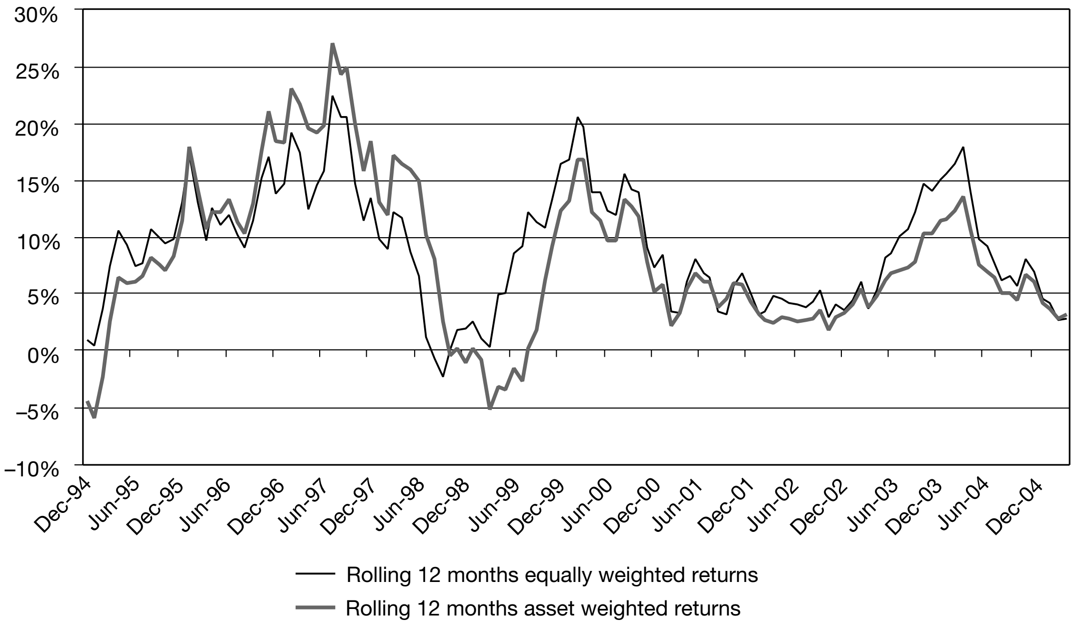

Figure 1 Equally vs asset weighted logarithmic returns

calculated for various time periods from January 1994 to April 2005. The results can be found in Table 3. The results are not subject to the survivorship bias, since both 'dead' and 'living' funds are included in the analysis. The instant history bias and the selection bias are hard to avoid and therefore affect the results. The survivorship bias of the TASS hedge fund database is calculated in the following subsection.

The annualised return differences between equally and asset weighted returns is -0.96 per cent over the 136-month time horizon. Interestingly, the difference is smaller in the time period from January 1994 to August 1999 than in the time period from September 1999 to April 2005. The return differences over the entire time period as well as the two sub-periods are not statistically significant at the 5 per cent significance level.

Figure 1 shows a comparison between the rolling 12-months equally weighted and the rolling 12-months asset weighted

Table 4 Survivorship bias based on a sample with 4,014 hedge funds[a]

	Jan 94–April 05	Sep 99–Apr 05	Jan 94–Aug 99
Equally weighted bias (%)	3.54	2.36	4.73
Asset weighted bias (%)	0.87	1.02	0.72
Differences in survivorship bias p.a. (%)	2.67	1.33	4.01
Standard deviation p.a. (%)	2.23	0.95	2.97
t-statistic	3.98	3.33	3.15

[a]The survivorship biases of equally weighted and asset weighted returns are calculated for a 136-month time horizon from January 1994 to April 2005 and for two sub-periods from January 1994 to August 1999 and from September 1999 to April 2005. The significance of the differences in the survivorship biases is tested with a *t*-statistic.

returns. Temporary differences between equally and asset weighted returns in the late 1990s can be observed. This phenomenon can be explained by the fact that very few large hedge funds were dominating the returns of the hedge fund industry at that time, drawing the asset weighted returns up in 1997 and 1998, but causing an underperformance in 1999.

Differences in the survivorship bias

The study also investigated whether smaller funds have a larger survivorship bias than larger funds. The results are illustrated in Table 4. The annualised differences between the survivorship bias calculated with equally weighted returns and the survivorship bias calculated with asset weighted returns is significant at the 1 per cent level for the 136-month period as well as for both sub-periods. It can therefore be stated that smaller hedge funds face an increased risk of exiting the database.

If one believes that asset weighted returns are more representative for the hedge fund industry, one needs to be careful in interpreting the results of many existing studies based on equally weighted returns. The findings based on asset weighted returns are useful from a macro perspective and relevant to measure the survivorship bias of the hedge fund industry in general. In order to interpret the results from the perspective of the individual hedge fund investor who is equally distributing his/her investments to a number of hedge funds regardless of the hedge fund size, the survivorship bias derived from equally weighted returns is the more appropriate measure.

Figure 2 illustrates the development of the survivorship bias in equally and asset weighted returns over time. Using 12-month rolling return data, one can see that the survivorship bias with asset weighted returns is less stable. Particularly in the period around 1999, the rolling survivorship bias with asset weighted returns is increasing significantly following the hedge fund crisis in 1998.

Funds may voluntarily stop reporting because they do not want to publish bad performance and harm their reputation. One cannot assume that funds dropping from the database are really ceasing their operation. TASS classifies exiting funds to some extent in seven categories. A summary can be found in Table 5. More than 50 per cent of the exiting funds have been liquidated and 3.3 per cent of the funds merged with other funds. Other reasons for exiting are more difficult to interpret. Sixty funds indicated that they closed for new investments, but the number might be much higher, since almost 44 per cent of the funds refused to state any reason for exiting or were unreachable.

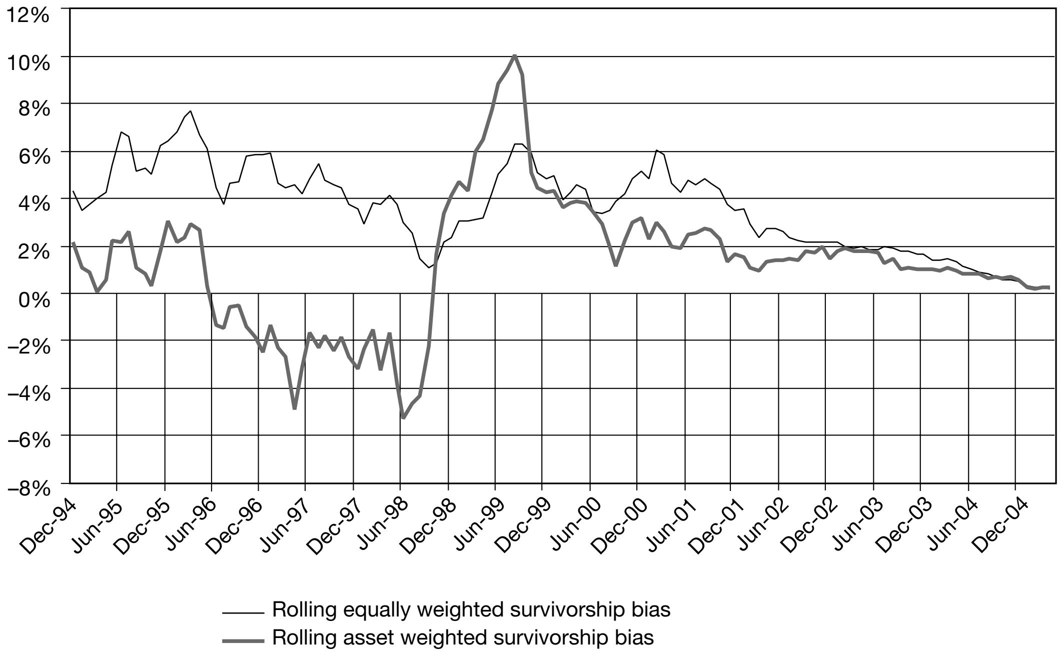

Figure 2 12-month rolling survivorship biases

Factor model

Equally weighted excess returns of hedge funds are explained by an asset class factor model in order to derive hedge fund alphas. The significance of each factor is first tested on a stand-alone basis, with 18 single factor models. The results can be found in Table 6. All the equity indices, both commodity indices, the gold index and the VIX are significant on a stand-alone basis at the 5 per cent significance level, but neither crude oil nor any of the bond indices are significant. The Wilshire Small Cap 1750 Index has the highest explanatory power. It is interesting to see that other equity indices including large cap and mid cap stocks are less useful to explain excess returns of hedge funds. This is an indication that hedge funds tend to focus on small cap stocks that have typically less research coverage from investment banks.

Table 7 represents the results of a multiple regression of 18 asset class factors on the returns of the sample consisting of 4,014 funds. Almost 55 per cent of the excess returns can be explained by the 18 asset class factor model. The Dow Jones AIG Commodity Index and the Merrill Lynch Treasury 10+ Year Index are significant at the 1 per cent significance level, while the MSCI Pacific and the MSCI Japan Index are significant at the 5 per cent significance level. The constant, indicating the alpha of hedge funds, is positive and statistically significant at the 10 per cent significance level. The annualised alpha of the model is 0.79 per cent. This result needs to be interpreted carefully, since hidden factors that are not captured in the 18 asset class factor model may explain a large portion of the alpha.

To avoid multicollinearities, the number of factors is reduced. Two different approaches are used: the first factor-reduction approach is looking for a model where all factors in the new

Table 5 TASS Graveyard database — reasons for exiting

Funds liquidated	1135
Funds no longer reporting to TASS	661
TASS has been unable to contact the manager for updated information	197
Unknown	133
Funds merged into another entity	74
Funds closed to new investment	60
Funds dormant	4
All Funds	2264

[a]The hedge funds in the TASS Graveyard database are classified according to their reasons for exiting. The classification takes data until April 2005 into account[a]

Table 6 Single factor models for 18 asset class factors

Asset classes	Factors	Factor beta	Standard error	*t*-statistic	Probability (%)
EQUITIES	WILSHIRE SMALL CAP 1750	0.176	0.020	8.831	0.00
	NASDAQ	0.108	0.015	7.447	0.00
	MSCI WORLD	0.205	0.029	7.103	0.00
	MSCI EUROPE	0.159	0.026	6.049	0.00
	WILSHIRE VALUE	0.180	0.028	6.342	0.00
	WILSHIRE GROWTH	0.148	0.023	6.547	0.00
	MSCI PACIFIC	0.128	0.024	5.391	0.00
	MSCI JAPAN	0.091	0.023	4.064	0.01
	WILSHIRE REIT	0.106	0.034	3.135	0.21
BONDS	JPM GL. GOV. BOND INDEX	0.090	0.075	1.194	23.44
	ML TREASURY 10+ YEAR	0.112	0.128	0.873	38.44
	LEHMAN HIGH YIELD INDEX	0.431	0.972	0.444	65.80
	LEHMAN BOND INDEX	0.436	1.469	0.297	76.72
COMMODITIES	DJ AIG COMMODITY INDEX	0.147	0.035	4.179	0.01
	GSCI	0.062	0.024	2.555	1.17
	GOLD INDEX	0.085	0.037	2.297	2.32
	IPE BRENT CRUDE OIL IND.	0.014	0.015	0.917	36.05
VOLATILITY	VIX	–0.031	0.008	–3.836	0.02

[a]A single asset class factor model for all 18 asset class factors is used to explain excess returns of hedge funds. Standard errors and *t*-statistics as well as *p* probabilities are calculated for each factor.

model are significant at the 5 per cent significance level and the maximum number of factors out of the 18 factors identified originally is used. It is found that the eight factor model in Table 8 gives the optimal solution to this approach. The asset class factor model with eight factors explains 51 per cent of the return variance. The adjusted *R*-squared of the eight factor model is greater than the adjusted *R*-squared of the 18 factor model.

The second factor-reduction approach starts with the condition of choosing one factor from each asset class, equities, bonds and commodities.[7] Since the number of factors is limited to three, one looks for the optimal combination of three factors of one equity, one bond and one commodity index in order to maximise the explanatory power of the model. The result is represented in Table 9. The combination of the Wilshire Small Cap 1750 Index, the Dow Jones AIG Commodity Index and the Merrill Lynch Treasury 10+ Year Index in a three factor model is explaining more than 45 per cent of the return variance.

The relevance of the three factors chosen in the three asset class factor model is further explored by breaking the sample data into 100 percentiles

Table 7 Asset class factor model with 18 factors[a]

Variable	Coefficient	Std. error	*t*-statistic	Prob. (%)
ALPHA	0.007	0.004	1.720	8.81
DJ AIG COMMODITY INDEX	0.185	0.064	2.871	0.49
ML TREASURY 10+ YEAR	0.420	0.149	2.824	0.56
MSCI PACIFIC	0.333	0.131	2.554	1.19
MSCI JAPAN	–0.242	0.105	–2.304	2.30
NASDAQ	0.115	0.059	1.951	5.34
WILSHIRE GROWTH	–0.141	0.100	–1.420	15.81
MSCI EUROPE	0.101	0.072	1.408	16.18
WILSHIRE SMALL CAP 1750	0.078	0.057	1.381	17.00
VIX	0.012	0.008	1.361	17.61
LEHMAN HIGH YIELD INDEX	–2.475	2.252	–1.099	27.40
GSCI	–0.052	0.049	–1.072	28.60
IPE BRENT CRUDE OIL IND.	–0.018	0.018	–0.963	33.73
LEHMAN BOND INDEX	3.067	3.288	0.933	35.29
GOLD INDEX	0.020	0.034	0.589	55.73
JPM GL. GOV. BOND INDEX	–0.061	0.114	–0.537	59.23
WILSHIRE REIT	–0.017	0.032	–0.529	59.80
WILSHIRE VALUE	0.039	0.074	0.522	60.23
MSCI WORLD	–0.094	0.185	–0.507	61.29
R-squared	0.545			
Adjusted *R*-squared	0.475			

[a]A multi-asset class factor model with 18 factors is used to explain equally weighted excess returns of hedge funds. Standard errors and *t*-statistics as well as *p*-probabilities are calculated for each factor.

Table 8 Asset class factor model with eight factors[a]

Variable	Coefficient	Std. error	*t*-statistic	Prob. (%)
ALPHA	0.003	0.001	2.831	0.54
NASDAQ	0.161	0.038	4.182	0.01
ML TREASURY 10+ YEAR	0.362	0.101	3.586	0.05
DJ AIG COMMODITY INDEX	0.199	0.058	3.439	0.08
MSCI EUROPE	0.105	0.035	2.977	0.35
WILSHIRE GROWTH	–0.189	0.064	–2.961	0.37
MSCI PACIFIC	0.274	0.111	2.457	1.54
MSCI JAPAN	–0.208	0.095	–2.202	2.95
GSCI	–0.079	0.038	–2.090	3.87
R-squared	0.510			
Adjusted *R*-squared	0.480			

[a]A multi-asset class factor model with eight factors is used to explain equally weighted excess returns of hedge funds. Standard errors and *t*-statistics as well as *p*-probabilities are calculated for each factor.

Table 9 Asset class factor model with three factors[a]

Variable	Coefficient	Std. error	*t*-statistic	Prob. (%)
ALPHA	0.002	0.001	2.276	2.44
WILSHIRE SMALL CAP 1750	0.176	0.020	8.939	0.00
DJ AIG COMMODITY INDEX	0.088	0.029	3.092	0.24
ML TREASURY 10+ YEAR	0.300	0.099	3.042	0.28
R-squared	0.452			
Adjusted *R*-squared	0.439			

[a]A multi-asset class factor model with three factors is used to explain equally weighted excess returns of hedge funds. Standard errors and *t*-statistics as well as *p*-probabilities are calculated for each factor.

Table 10 Principal component analysis[a]

Component	% of variance explained	Cumulative % of variance explained
1	47.05	47.05
2	4.64	51.68
3	2.29	53.98
4	2.05	56.03
5	1.95	57.98
6	1.81	59.79
7	1.69	61.48
8	1.62	63.10
9	1.47	64.58
10	1.46	66.03

[a]The funds are ranked according to their fund sizes and 100 asset percentiles are built in each month. The principal component analysis is based on the excess returns of the 100 percentiles. The first component explains 47.05% of the variance, and the first ten components explain 66.03% of the variance.

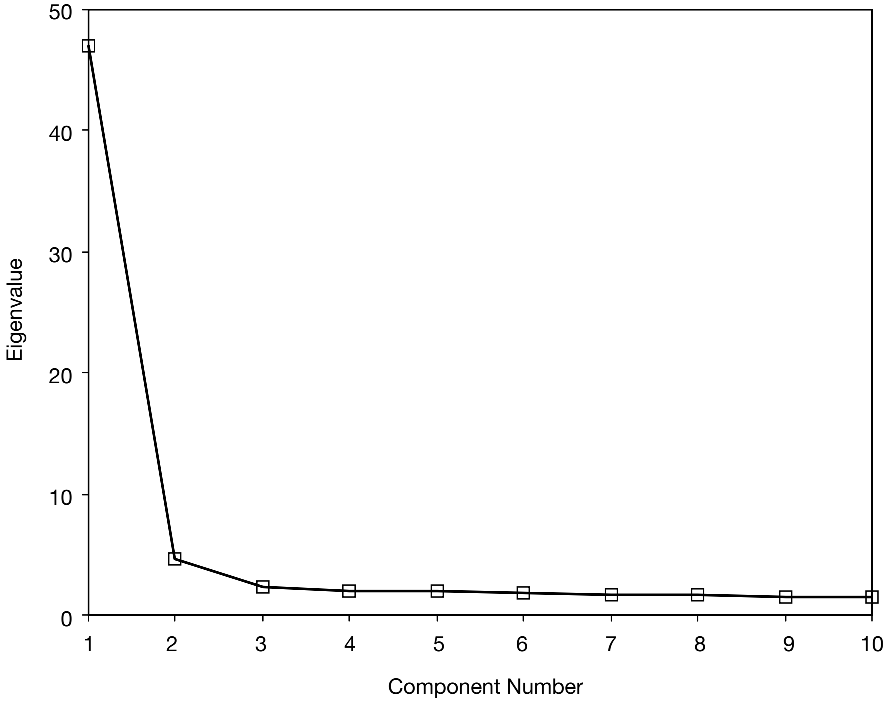

Figure 3 Scree plot of principal component analysis

according to their fund sizes and conducting a principal component analysis on the 100 time series representing the average returns of each percentile. The result of the principal component analysis is presented in Table 10. The first component explains 47.05 per cent of the variance, and the first ten components together explain 66.03 per cent of the variance. The cumulative variance explained of the first three components is 53.98 per cent and therefore higher than the variance explained by the asset class factor model with three factors shown in Table 9. The scree plot of the principal component analysis presented in Figure 3 illustrates the dominance of the first component versus the other components based on the eigenvalues.

Table 11 Hedge fund assets and returns[a]

Percentile	Average assets ($)	Average returns (%)
91st–100th	603,601,638	6.53
81st–90th	132,397,013	6.90
71st–80th	68,163,293	7.63
61st–70th	40,035,580	8.25
51st–60th	24,696,784	8.69
41st–50th	15,176,532	9.95
31st–40th	8,965,755	9.52
21st–30th	4,961,530	10.34
11th–20th	2,333,363	8.91
1st–10th	642,271	7.28

[a]The sample of 4,014 hedge funds is classified in percentiles according to their fund sizes. The second column illustrates the average fund sizes of each decile. The third column shows the average returns for each decile.

Cross-sectional regressions

This sub-section uses cross-sectional regressions to identify the impact of fund sizes on excess returns, standard deviations, Sharpe ratios and alphas. The breakdown of the sample data into 100 percentiles is used, based on their fund sizes. Each percentile can be regarded as a sub-sample. The constitution of the sub-samples changes in each month as the assets under management are changing, and funds with increasing assets relative to its peer group fall into a higher percentile. The first regression relates the logarithm of the average fund sizes of the 100 sub-samples to the average excess returns of the sub-samples.

Table 11 shows the deciles of fund sizes and the annualised average returns of the hedge funds in each decile. Very small funds of the bottom decile are underperforming. This phenomenon can be explained by an economy of scale effect. The operational expenses play a significant role for smaller funds and make it uneconomic to run a fund with a very small asset base. Funds from the 21st to the 50th percentile have the best performance. These hedge funds are still relatively small, with an asset base of less than US$20m. Many institutional investors are focusing on hedge funds with a larger asset base and are therefore eliminating funds with the highest return potential based on the indication of asset sizes. An institutional investor who is looking for funds with a minimum of US$50m under management will only focus on funds in the top three deciles. It is known that many of the largest funds are closed for investment, and many investors are therefore left with a relatively small universe of a few hundred hedge funds. Funds above the 50th asset percentile show a negative relationship between asset size and performance.

The relationship between fund sizes and returns is further investigated. Therefore, a simple regression analysis of the logarithm of the fund sizes is applied to the average returns of the asset percentiles. The results of the linear regression analysis are presented in Figure 4 and Table 12. Each data point in Figure 4 represents an average annualised return for one asset percentile in the data sample. The linear relationship between returns and fund sizes is negative and statistically significant at the 5 per cent significance level.

Investigation of the scatter plot in Figure 4 suggests a non-linear relationship between fund sizes and returns. Applying a quadratic regression analysis, one can find a concave relationship that basically confirms the finding of Getmansky (2004). The curve

Table 12 Regression results of fund sizes versus annualised returns[a]

Dependent variable Independent variables	Annualised returns Coefficient	Std. error	*t*-statistic	Prob.
Linear regression				
C	0.1310	0.0197	6.6569	0.0000
Log(Assets)	–0.0028	0.0012	–2.3816	0.0192
R-squared	0.0547			
Adjusted *R*-squared	0.0451			
Quadratic regression				
C	–0.2670	0.1129-	–2.3645	0.0200
Log(Assets)	0.0462	0.0137	3.3590	0.0011
Log(Assets)^2	–0.0015	0.0004	–3.5737	0.0006
R-squared	0.1647			
Adjusted *R*-squared	0.1475			

[a]The funds are ranked according to their fund sizes, and 100 asset percentiles are built in each month. In the linear regression, the logarithm of the average assets of each of the 100 percentiles are regressed on the average annualised returns. In the linear regression, the average annualised returns are regressed on the logarithms of the average assets of each of the 100 percentiles. In the quadratic regression, the average annualised returns are regressed on the logarithms of the average assets of each of the 100 percentiles and on the squared logarithms of the assets.

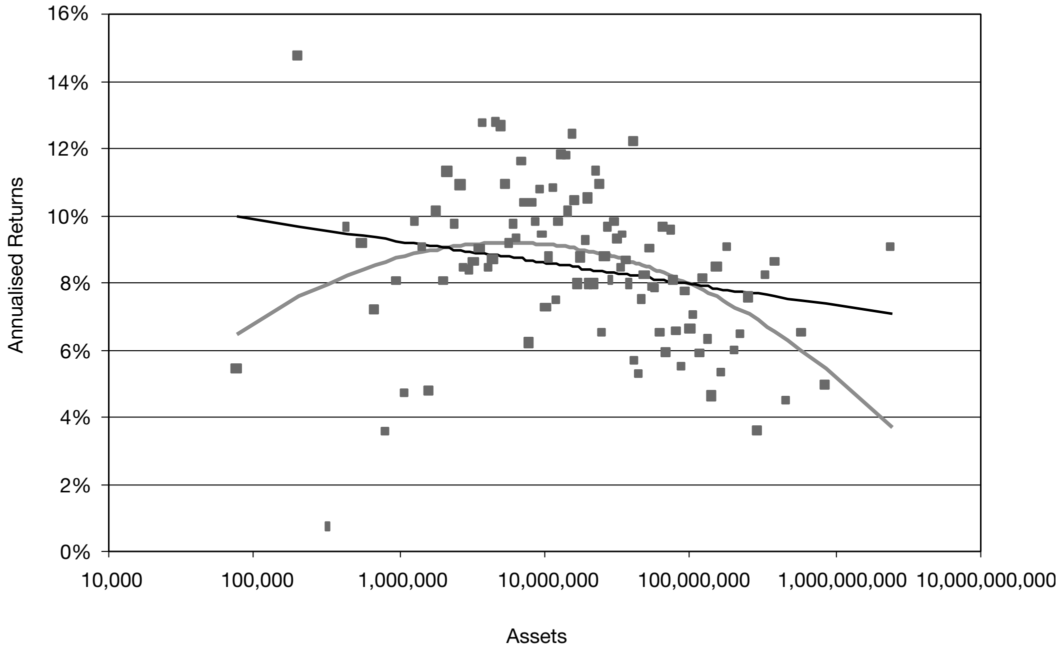

Figure 4 Log of asset sizes vs annualised returns

in Figure 4 represents the results of the quadratic regression. The quadratic term in the regression analysis is significant at the 1 per cent significance level.

One can see that, in particular, very small funds with less than US$1,000,000 in assets under management are sometimes underperforming. The underperformance can be explained by higher relative operational costs. Minimum fixed costs for the management of the hedge fund vehicle, fund administration and custody

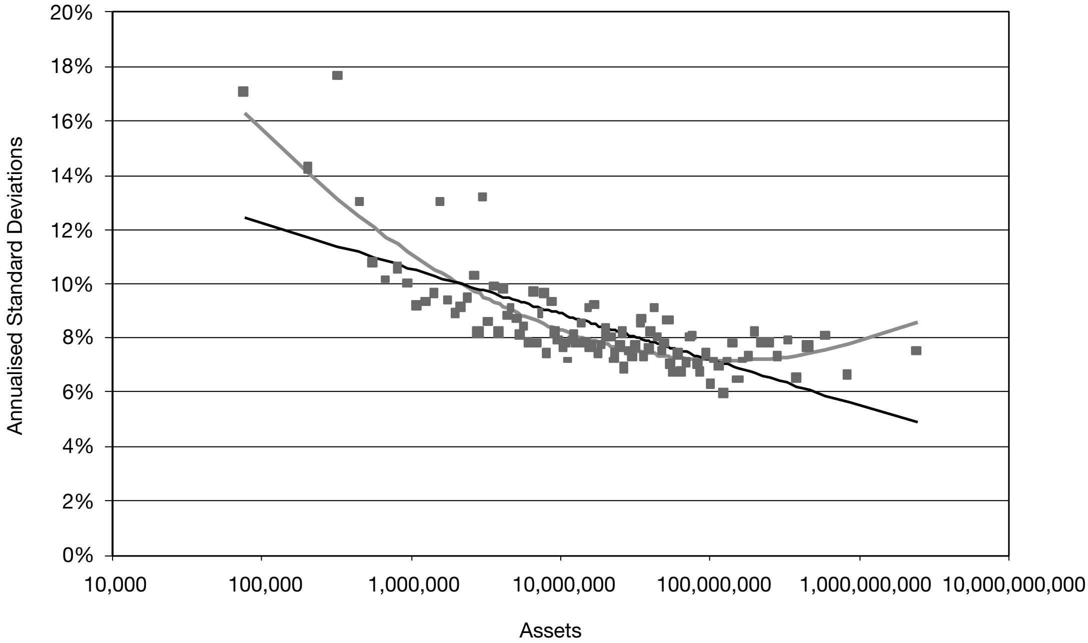

Figure 5 Log of asset sizes vs annualised standard deviations

are the major operational costs that are diminishing net returns for the investor. Generally, one can observe diminishing returns of larger funds that go hand in hand with a lower risk appetite of large funds owing to a more institutional client base.

The impact of fund sizes on volatilities in a linear and quadratic regression analysis is also explored. The volatilities refer to percentiles and therefore portfolios of hedge funds, rather than individual hedge funds. The volatility of portfolios of hedge funds is generally lower than the volatility of individual hedge funds, owing to diversification effects.

The linear regression of fund sizes on volatilities indicates a significant relationship at the 1 per cent significance level. The relationship is also tested for convexity, and it is found that the quadratic term in the quadratic regression is also statistically significant at the 1 per cent significance level. The results can be found in Table 13 and Figure 5.

The relationship between fund sizes and standard deviations is intuitive, since large funds generally benefit from a broader diversification and therefore a reduction in volatilities. Larger funds very often attracted assets based on a proven track record and might therefore shift their focus on capital preservation. Highly aggressive strategies with concentrated bets are sometimes more difficult to implement with a large asset base, owing to capacity constraints.

Larger funds are very often in a position to control the asset flows better, and therefore benefit from a more stable income. To control the asset flows, larger funds can more easily afford less favourable liquidity conditions for investors and keep investors in the fund by applying lockup periods, redemption gates or redemption fees for early withdrawal of investments. A stable asset base allows for better planning of investments. The manager can therefore more consistently apply his/her strategy or also invest in illiquid securities that

Table 13 Regression results of fund sizes versus standard deviations[a]

Dependent variable Independent variables	Annualised standard deviations Coefficient	Std. error	t-statistic	Prob.
Linear regression				
C	0.2061	0.0113	18.2525	0.0000
Log(Assets)	−0.0073	0.0007	−10.8263	0.0000
R-squared	0.5446			
Adjusted R-squared	0.5400			
Quadratic regression				
C	0.6431	0.0522	12.3194	0.0000
Log(Assets)	−0.0611	0.0064	−9.6051	0.0000
Log(Assets)^2	0.0016	0.0002	8.4871	0.0000
R-squared	0.7387			
Adjusted R-squared	0.7333			

[a]The funds are ranked according to their fund sizes, and 100 asset percentiles are built in each month. In the linear regression, the standard deviations are regressed on the logarithms of the average assets of each of the 100 percentiles. In the quadratic regression, the standard deviations are regressed on the logarithms of the average assets of each of the 100 percentiles and on the squared logarithms of the assets.

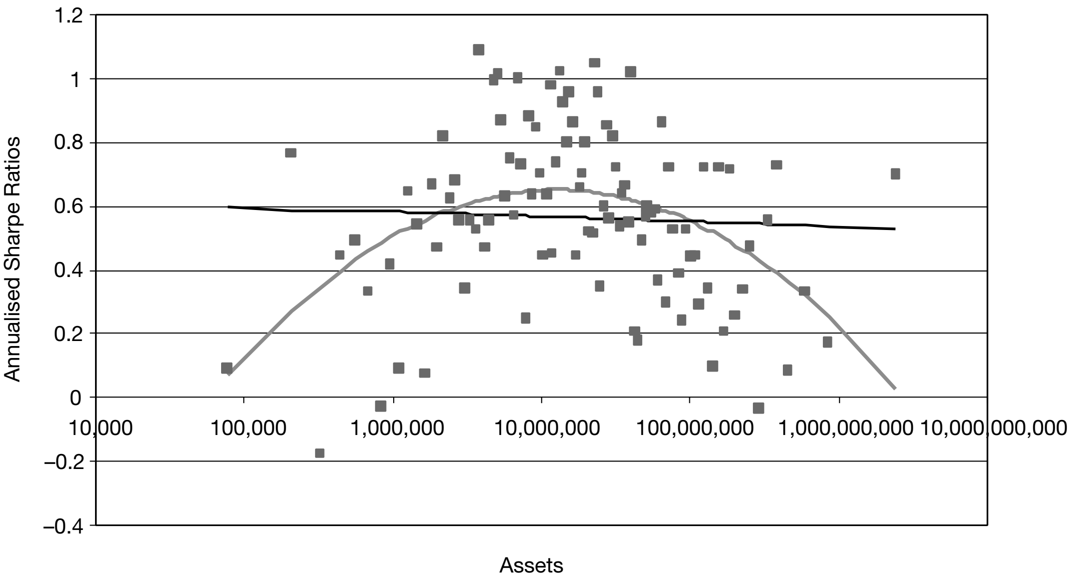

Figure 6 Log of asset sizes vs annualised Sharpe ratios

are not priced on a daily basis and therefore diminish the volatility of the fund.

Next, the relationship between fund sizes and Sharpe ratios are explored with a linear and a quadratic regression approach. Similar to standard deviations, the Sharpe ratios refer to asset percentiles representing portfolios of hedge funds of a similar size. Owing to the diversification effect of portfolios, the Sharpe ratios of individual hedge funds are typically lower than the Sharpe ratios of portfolios. The findings of the regression analysis are illustrated in Figure 6 and Table 14. In the linear regression, the relationship between fund sizes and Sharpe ratios is slightly negative, but not statistically significant.

Table 14 Regression results of fund sizes versus Sharpe ratios[a]

Dependent variable Independent variables	Annualised Sharpe ratios Coefficient	Std. error	*t*-statistic	Prob.
Linear regression				
C	0.6673	0.2358	2.8303	0.0056
Log(Assets)	–0.0062	0.0140	–0.4427	0.6590
R-squared	0.0020			
Adjusted *R*-squared	0.0082			
Quadratic regression				
C	–5.2874	1.3018	–4.0617	0.0001
Log(Assets)	0.7265	0.1585	4.5833	0.0000
Log(Assets)^2	–0.0222	0.0048	–4.6376	0.0000
R-squared	0.1831			
Adjusted *R*-squared	0.1663			

[a]The funds are ranked according to their fund sizes, and 100 asset percentiles are built in each month. In the linear regression, the Sharpe ratios are regressed on the logarithms of the average assets of each of the 100 percentiles. In the quadratic regression, the Sharpe ratios are regressed on the logarithms of the average assets of each of the 100 percentiles and on the squared logarithms of the assets.

Table 15 Regression results of fund sizes versus annualised alphas[a]

Dependent variable Independent variables	Annualised alphas (3-factor model) Coefficient	Std. error	*t*-statistic	Prob.
Linear regression				
C	0.0763	0.0203	3.7581	0.0003
Log(Assets)	–0.0029	0.0012	–2.3580	0.0204
R-squared	0.0537			
Adjusted *R*-squared	0.0440			
Quadratic regression				
C	–0.2998	0.1177	–2.5461	0.0125
Log(Assets)	0.0434	0.0143	3.0291	0.0031
Log(Assets)^2	–0.0014	0.0004	–3.2386	0.0016
R-squared	0.1460			
Adjusted *R*-squared	0.1284			

[a]The alphas are derived from hedge fund returns and a multi-asset class factor model with three factors. The factors are the Dow Jones AIG Commodity Index, the Merrill Lynch Treasure 10+ Year Index and the Wilshire Small Cap 1750 Index. For the regression analysis, the funds are ranked according to their fund sizes, and 100 asset percentiles are built in each month. In the linear regression, the alphas derived from the three-factor model are regressed on the logarithms of the average assets of each of the 100 percentiles. In the quadratic regression, the alphas are regressed on the logarithms of the average assets of each of the 100 percentiles and on the squared logarithms of the assets.

In the quadratic regression, the quadratic term is significant at the one per cent level. Using the results of the quadratic regression, it is therefore generally possible to find an optimal fund size with the highest Sharpe ratio, although the quadratic relationship is not necessarily obvious from the scatter plot. Additionally, as shown in Figure 6, non-linearity tends to be driven by very small funds.

The next step explores the relationship between fund sizes and the alphas derived from the asset class factor model with three factors. Recall that the alphas in the three factor model account for risk exposures to commodities, small-cap stocks and bonds. Figure 7 and Table 15 illustrate the results of the regression analysis. The coefficient of the linear regression is negative and significant at the 5 per cent significance level. This result indicates lower alphas for larger hedge funds. The coefficients of the

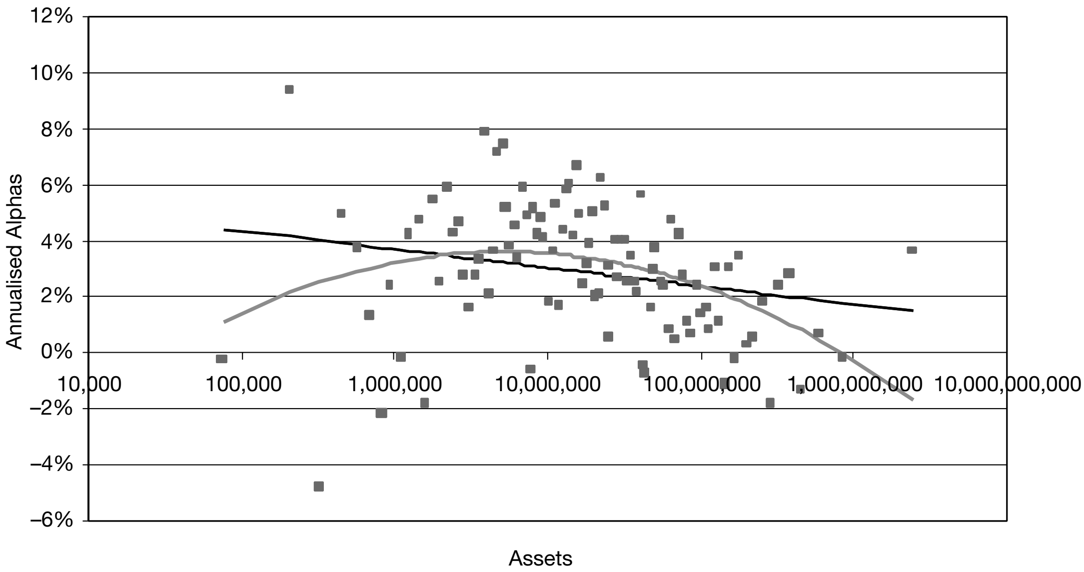

Figure 7 Log of asset sizes vs annualised alphas.

quadratic regression are significant and indicate a concave relationship between fund sizes and alphas.

It remains difficult to estimate whether large funds are taking more exposure to risk factors that are not captured by the simple asset class factor model compared with small funds. In any case, the higher relative expense ratio of very small funds is decreasing the alphas based on net returns. Therefore, very small funds also have more frequently negative alphas.

Based on the parameters found in the quadratic regression analysis, one can derive optimal fund sizes. The convex and concave relationships tend to be driven by outliers. The usefulness of the quadratic regressions is therefore limited.

Conclusion

This paper contributes to the existing literature about hedge fund performance with a detailed analysis of the impact of fund sizes on returns, Sharpe ratios and alphas derived from a multi-asset class factor model. Based on a large sample of hedge fund returns, empirical evidence is revealed for a negative relationship between fund sizes and returns, using cross-sectional regression techniques, but it is also found that particularly very small funds are underperforming on average. It is conjectured that the underperformance of very small funds is based on the higher total expense ratios. A negative relationship between standard deviations and fund sizes is also observed. Generally, larger funds tend to have lower volatilities but similar Sharpe ratios. Very small funds have a clear disadvantage to compete with medium- and larger-sized funds.

It is shown that, in the long term, the average alphas generated by hedge funds derived from a three-factor, eight-factor and 18-factor model are statistically significant at the 10 per cent significance level. Similar to the analysis based on annualised returns, the analysis based on three-factor alphas reveals that smaller funds are outperforming on average.

Hedge fund managers are primarily remunerated with a performance fee. In absolute terms, the performance fee can be increased by higher returns, but also by a larger asset base. A hedge fund manager who is maximising his income may therefore be willing to grow a fund above its optimal size from a pure performance perspective. In the long term, a good performance is instrumental in attracting assets and also enhances the reputation of the manager. Therefore, the manager faces a trade-off between optimising the performance of the fund and optimising his revenues. Nevertheless, the empirical evidence for managers increasing their fund size beyond the optimal point is hard to prove, since the number of large funds exceeding US$100m assets under management is small compared with the total number of hedge funds.

Different hedge fund strategies have different capacity limits. The strategy-specific characteristics of the asset-return relationship open opportunities for further research projects. For the percentiles-based approach, however, the number of hedge funds available for each hedge fund strategy is not sufficient to break the strategy-specific samples further down into 100 sub-samples over a 136-month period. The results of strategy-specific analysis are therefore limited by the data available.

Notes

1. Survivorship bias occurs if the database only contains information on 'surviving funds'. Following Malkiel's (1995) method, the bias is evaluated via the difference in the performance of the 'observable' portfolio (investment in each fund in the database from the beginning of the data sample) and the portfolio of surviving funds.
2. Owing to insufficient return data 336 hedge funds were eliminated. Thirty-six funds have more than 10 per cent missing return data in the time period from January 1993 to April 2005, and the remaining 300 funds do not report at all return data in the sample time period.
3. Owing to insufficient asset data, 1,357 hedge funds were eliminated.
4. Schneeweis and Kazemi (2003) investigate three different methods to explain excess returns: (a) a single-factor approach using a small capitalisation equity index; (b) a multi-factor linear unconditional model; and (c) a SDF/GMM approach. The authors find that, in most cases, the alphas are rather similar regardless of the empirical methodology applied.
5. The risk-free interest rate is discounted from all asset class factors with the exception of the CBOE SPX Volatility Index, since the volatility index is the only index that is not representing an asset class in a traditional sense.
6. The CSFB-Tremont Hedge Fund Indices contain approximately 400 hedge funds and represents US$160bn in assets. The index clearly focuses on large hedge funds. Hedge funds require a minimum of US$10m assets under management, a minimum track record of one year and audited financial statements in order to become part of the index.
7. Volatility was dropped as an asset class, since the CBOE SPX Volatility Index is not significant in a four-asset class factor model in combination with the other three asset classes.

References

Agarwal, V. and Naik, N. (2000) 'Performance Evaluation of Hedge Funds with Option-based and Buy-and-Hold Strategies', Working Paper, London Business School.

Agarwal, V. and Naik, N. (2001) 'Characterizing Hedge Fund Strategies with Buy-and-Hold and Option-based Strategies', Working Paper, London Business School

Agarwal, N. and Naik, N. (2003) 'Risks and Portfolio Decisions Involving Hedge Funds', Working Paper, London Business School.

Amenc, M., Curtis, S. and Martellini, L. (2003) 'The Alpha and Omega of Hedge Fund Performance', Working Paper, Edhec/USC.

Brealey, R. A. and Kaplanis, E. (2001) 'Changes in Factor Exposures of Hedge Funds', Working Paper.

Clark, A. (2003) 'Does Fund Size Affect Performance?', *Lipper Research Study*.

Edwards, F. R. and Caglayan, M. (2001) 'Hedge Fund Performance and Manager Skill', *Journal of Futures Markets*, 21(11), 1003–28.

Fama, E. F. and French, K. R. (1993) 'Common Risk Factors in the Returns on Stocks and Bonds', *Journal of Financial Economics*, 33(1), 3–56.

Fama, E. F. and French, K. R. (1996) 'Multi-factor Explanations of Asset Pricing Anomalies', *Journal of Finance*, 51(1), 131–55.

Fung, W. and Hsieh, D. (1997) 'Empirical Characteristics of Dynamic Trading Strategies: The Case of Hedge Funds', *The Review of Financial Studies*, 10(2), 275–302.

Getmansky, M. (2004) 'The Life Cycle of Hedge Funds: Fund Flows, Size and Performance', Working Paper, MIT.

Goetzmann, W., Ingersoll, J. and Ross, S. (2003) 'High Water Marks and Hedge Fund Management Contracts', *Journal of Finance*, 58(4), 1685–717.

Gregoriou, G. N. and Rouah, F. (2003) 'Large versus Small Hedge Funds: Does Size Affect Performance?', *Journal of Alternative Investments*, 5(5), 75–7.

Hedges, R. J. (2003) 'Size vs. Performance in the Hedge Fund Industry', *Journal of Financial Transformation*, 10, 14–17.

Herzberg M. M. and Mozes, H. A. (2003) 'The Persistence of Hedge Fund Risk: Evidence and Implications for Investors', *Journal of Alternative Investments*, 6(2), 22–42.

Kazemi, H. and Schneeweis, T. (2003) 'Conditional Performance of Hedge Funds', Working Paper, CISDM, University of Massachusetts.

Liang, B. (1999) 'On the Performance of Hedge Funds', *Financial Analysts Journal*, 55(4), 72–85.

Liang, B. (2000) 'Hedge Funds: The Living and the Dead', *Journal of Financial and Quantitative Analysis*, 35(3), 309–25.

Liang, B. (2003) 'The Accuracy of Hedge Fund Returns', *The Journal of Portfolio Management*, 29(3), 111–22.

Malkiel, B. (1995) 'Returns from Investing in Equity Mutual Funds 1971 to 1991', *Journal of Finance*, 50(2), 549–72.

Schneeweis, T., Kazemi, H. and Martin, G. (2003) 'Understanding Hedge Fund Performance: Research Issues Revisited — Part II', *The Journal of Alternative Investments*, 5(4), 8–30.

Sharpe, W. F. (1992) 'Asset Allocation: Management Style and Performance Measurement', *The Journal of Portfolio Management*, 18(2), 7–19.